

Problem Set 1  
Complex Numbers and Polynomials

(1)

Prove the following properties of complex numbers:

$$\begin{aligned} a. \quad \overline{z+w} &= \bar{z} + \bar{w} \\ b. \quad \overline{z\bar{w}} &= \bar{z} \cdot w \\ c. \quad \overline{(z/w)} &= \bar{z}/\bar{w} \end{aligned}$$

(2)

Let  $r, s,$  and  $t$  be the roots of  $x^3 - 6x^2 + 5x - 7 = 0$ . Find

$$\frac{1}{r^2} + \frac{1}{s^2} + \frac{1}{t^2}$$

Hint: Remember Vieta's...

(3)

List the roots of  $x^8 - 1 = 0$  with positive real part.

(4)

List the roots of  $x^4 + x^2 + 1 = 0$

(5)

A function  $f$  is defined on the complex numbers by  $f(z) = (a + bi)z$ , where  $a$  and  $b$  are positive numbers. This function has the property that the image of each point in the complex plane is equidistant from that point and the origin. Given that  $|a + bi| = 8$  and that  $b^2 = m/n$ , where  $m$  and  $n$  are relatively prime positive integers. Find  $m + n$ .

Challenge questions

(1)

Compute the area of the region defined by the roots of the polynomial  $x^7 + x^6 + x^5 + \dots + 1 = 0$  in the complex plane.

(2)

Find the polynomial whose roots are the reciprocals of the roots of  $x^4 - 3x^2 + x - 9 = 0$ .

Hint: If you're trying to find the actual roots, you're doin' it wrong.

(3)

For those that are taking Calculus, find the integral  $\int x e^x \sin(x) dx$ .

Hint: Remember Euler's formula ... also, you might want to look up integration by parts.

(4)

An unfair coin with probability  $\frac{1}{3}$  of heads and probability  $\frac{2}{3}$  of tails is flipped 50 times. Find the probability, in closed form, of obtaining an even number of heads.